Inter (Part-I) 2017

Mathematics	Group-I	PAPER: I
Time: 2.30 Hours	(SUBJECTIVE TYPE)	Marks: 80

SECTION-I

2. Write short answers to any EIGHT (8) questions: (16)

(i) Does the set {0, –1} possess closure property with respect to:

(a) addition (b) multiplication

Ans

(a)

Since $(-1) + (-1) = -2 \notin \{0, -1\}$,

so {0, –1} is not closed w.r.t addition.

(b)

Since $(-1) \times (-1) = 1 \notin \{0, -1\}$,

So {0, –1} is not closed w.r.t multiplication.

(ii) Find multiplication inverse of a + bi.

Ans For multiplicative inverse, the reciprocal of a and b is:

$$\frac{1}{(a)^2 + (b)^2} + \frac{-b}{(a)^2 + (b)^2} i = \frac{1}{a^2 + b^2} - \frac{b}{a^2 + b^2} i$$

(iii) Prove that $|z_1 z_2| = |z_1| |z_2| \ \forall z_1, z_2 \in C$

Ans $\text{L.H.S} = |z_1 z_2|$

As we known that:

$z_1 = a + ib, z_2 = c + id$, then

$$\begin{aligned}
|z_1, z_2| &= |(a + ib)(c + id)| \\
&= |(ac - bd) + (ad + bc) i| \\
&= \sqrt{(ac - bd)^2 + (ad + bc)^2} \\
&= \sqrt{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2} \\
&= \sqrt{(a^2 + b^2)(c^2 + d^2)} \\
&= |z_1| \cdot |z_2|
\end{aligned}$$

This result may be stated thus:

The modulus of the product of two complex numbers is equal to the product of their moduli.

(iv) Define proper subset and improper subset.

Ans **Proper Subset:**

If A is a subset of B and B contains at least one element which is not an element of A, then A is said to be a proper subset of B. In such a case, we write:

$A \subset B$ (A is a proper subset of B)

Improper Subset:

If A is subset of B and A = B, then we say that A is an improper subset of B. From this definition, it also follows that every set A is an improper subset of itself.

(v) Show that the statement is tautology $\sim(p \to q) \to p$.

Ans

p	q	$p \to q$	$\sim(p \to q)$	$\sim(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

Since all the possible values of $\sim(p \to q) \to p$ are true. Thus $\sim(p \to q) \to p$ is a tautology.

(vi) If (G, $\dot{\times}$) is a group with 'e' its identity then 'e' is unique?

Ans Suppose the contrary that identity is not unique. And let e′ be another identity.

e, e′ being identities, we have

$e' \dot{\times} e = e \dot{\times} e' = e'$ (e is an identity) (i)

$e' \dot{\times} e = e \dot{\times} e' = e$ (e′ is an identity) (ii)

By comparing (i) and (ii), we get

$e' = e$

Thus the identity of a group is always unique.

(vii) $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, show that $A^4 = I_2$.

Ans

$$A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$A^2 = A.A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} i(i) + 0(1) & i(0) + 0(-i) \\ 1(i) + (-i)(1) & 1(0) + (-i)(-i) \end{bmatrix}$$

$$= \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = A^2 . A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1(-1) + 0(0) & -1(0) + 0(-1) \\ 0(-1) + (-1)(0) & 0(0) + (-1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$A^4 = I_2$ Proved

(viii) $A = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$, **show that** $A - (\bar{A})^t$ **is skew-hermitian.**

Ans Given, $A = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$

$$\bar{A} = \begin{bmatrix} -i & 1-i \\ 1 & i \end{bmatrix}$$

$$(\bar{A})^t = \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix}$$

Let, $Y = A - (\bar{A})^t$

$$= \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix} - \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix} = \begin{bmatrix} 2i & i \\ i & -2i \end{bmatrix}$$

Now, $(\bar{Y})^t = \begin{bmatrix} -2i & -i \\ -i & 2i \end{bmatrix}^t$

$$= \begin{bmatrix} -2i & -i \\ -i & 2i \end{bmatrix} = -Y$$

Thus $Y = A - (\bar{A})^t$ is skew-hermitian.

(ix) **Without expansion show that** $\begin{vmatrix} bc & ca & ab \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ a & b & c \end{vmatrix} = 0.$

Ans $\text{L.H.S} = \begin{vmatrix} bc & ca & ab \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ a & b & c \end{vmatrix}$

Multiplying all elements of second row by 'abc', we have

$$= \frac{1}{abc}\begin{vmatrix} \frac{bc}{abc} & \frac{ca}{abc} & \frac{ab}{abc} \\ a & b & c \\ a & b & c \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} bc & ca & ab \\ bc & ca & ab \\ a & b & c \end{vmatrix}$$

Since all elements of 1^{st} row and 2^{nd} row are identical, so

$$= \frac{1}{abc}(0) = 0 = \text{R.H.S}$$

(x) Solve the equation $x^{1/2} - x^{1/4} - 6 = 0$.

Ans This given equation can be written as:

$$(x^{1/4})^2 - x^{1/4} - 6 = 0$$

Let $x^{1/4} = y$

$\therefore$ The given equation becomes

$$y^2 - y - 6 = 0$$

$$(y - 3)(y + 2) = 0$$

$$y - 3 = 0 \quad ; \quad y + 2 = 0$$

$$y = 3 \quad ; \quad y = -2$$

As $x^{1/4} = y$

So, $x^{1/4} = 3$

$$(x^{1/4})^4 = (3)^4$$

$$x = 81$$

and $x^{1/4} = y$

$$x^{1/4} = -2$$

$$(x^{1/4})^4 = (-2)^4$$

$$x = 16$$

Hence the solution set is $\{16, 81\}$

(xi) When $x^3 + kx^2 - 7x + 6$ is divided by $x + 2$ the remainder is -4? Find the value of k.

Ans Let $f(x) = x^3 + kx^2 - 7x + 6$

and $x - a = x + 2$, we have

$$a = -2$$

(By Remainder Theorem)

Remainder $= f(-2)$

$$= (-2)^3 + k(-2)^2 - 7(-2) + 6$$

$$= -8 + 4k + 14 + 6$$

$$= 4k + 12$$

Given that remainder = –4

$\therefore \quad 4k + 12 = -4$

$4k = -4 - 12$

$4k = -16$

$k = -4$

(xii) Prove that $1 + \omega + \omega^2 = 0$.

Ans We know that cube roots of unity are:

$$1, \frac{-1+\sqrt{3}i}{2} \text{ and } \frac{-1-\sqrt{3}i}{2}$$

If $\quad \omega = \dfrac{-1+\sqrt{3}i}{2}$,

then $\omega^2 = \dfrac{-1-\sqrt{3}i}{2}$

Sum of all the three cube roots

$$1 + \omega + \omega^2 = 1 + \frac{-1+\sqrt{3}i}{2} + \frac{-1-\sqrt{3}i}{2}$$

$$= \frac{2-1+\sqrt{3}i-1-\sqrt{3}i}{2}$$

$$= \frac{0}{2} = 0$$

Hence sum of cube roots of unity

$1 + \omega + \omega^2 = 0$

3. Write short answers to any EIGHT (8) questions: (16)

(i) Resolve $\dfrac{1}{x^2-1}$ into partial fractions.

Ans

$$\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)}$$

$$\frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$1 = A(x-1) + B(x+1) \qquad (1)$

Put $\quad x + 1 = 0$

$1 = -2A$

$A = -\dfrac{1}{2}$

Now put $x - 1 = 0$

$x = 1$ in (1), we get.

$1 = 2B$

$B = \dfrac{1}{2}$

Now,

$$\frac{1}{(x+1)(x-1)}=\frac{-\frac{1}{2}}{x+1}+\frac{\frac{1}{2}}{x-1}$$

$$=-\frac{1}{2(x+1)}+\frac{1}{2(x-1)}$$

Which are required partial fractions.

(ii) **If $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in G.P, show that $r=\pm\sqrt{\frac{a}{c}}$.**

Ans Given $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in G.P

Let r be the common ratio of the G.P

$$\therefore \quad r=\frac{\frac{1}{b}}{\frac{1}{a}}=\frac{a}{b} \qquad \text{(i)}$$

$$\text{Also} \quad r=\frac{\frac{1}{c}}{\frac{1}{b}}=\frac{b}{c} \qquad \text{(ii)}$$

Multiply (i) and (ii),

$$r^2=\frac{a}{b}\times\frac{b}{c}$$

$$r=\pm\sqrt{\frac{a}{c}}$$

(iii) **Convert recurring decimal 0.7 into vulgar fraction.**

Ans

$$0.7=0.7777\ \ldots\ldots$$
$$=0.7+0.07+0.007+0.0007+\ldots\ldots$$
$$=\frac{7}{10}+\frac{7}{100}+\frac{7}{1000}+\frac{7}{10,000}+\ldots\ldots$$

Here, $a=\frac{7}{10}$,

$$r=\frac{7}{100}\times\frac{10}{7}=\frac{1}{10}$$

$$\therefore \quad S_\infty=\frac{\left(\frac{7}{10}\right)}{1-\frac{1}{10}}=\frac{7}{9}$$

(iv) If 5 is the harmonic mean between 2 and b, find b?

Ans Here, $a = 2, b = b$

We know that

$$H.M = \frac{2ab}{a+b}$$

By given condition,

$$\Rightarrow \quad H.M = \frac{2(2)(b)}{2+b} = 5$$

$$\Rightarrow \quad \frac{4b}{2+b} = 5$$

$$4b = 5(2+b)$$

$$4b = 10 + 5b$$

$$4b - 5b = 10$$

$$-b = 10$$

$$\boxed{b = -10}$$

(v) Find the A.P. whose nth term is 3n – 1.

Ans Given, $a_n = 3n - 1$

Substituting n = 1, 2, 3, 4 and so on.

For $n = 1$	$a_1 = 3(1) - 1 = 2$
$n = 2$	$a_2 = 3(2) - 1 = 5$
$n = 3$	$a_3 = 3(3) - 1 = 8$
$n = 4$	$a_4 = 3(4) - 1 = 11$

and so on.

Therefore, the required A.P is 2, 5, 8, 11, ... , $3n - 1$.

(vi) How many words can be formed from the letters of the word 'Objective' using all letters without repeating any one?

Ans We have to form permutation of 9 letters taken 9 at a time.

$$^9P_9 = 9!$$

$$= 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 362,880$$

(vii) In how many ways 4 keys can be arranged on a circular key ring?

Ans 4 keys can be arranged on a circular key ring in $\frac{1}{2}(3!)$ or 3 ways.

(viii) Find the values of n and r when $^nC_r = 35$ and $^nP_r = 210$.

Ans $^nC_r = 35$

$$\frac{n!}{(n-r)!\,r!} = 35 \quad (1)$$

$^nP_r = 210$

$$\frac{n!}{(n-r)!} = 210 \quad (2)$$

Using eq. (2) in eq. (1),

$$\frac{210}{r!} = 35$$

$$\Rightarrow \quad r! = \frac{210}{35}$$

$$r! = 6$$

$$r! = 3!$$

$$\boxed{r = 3}$$

Put in (2),

$$\frac{n!}{(n-3)!} = 210$$

$$\frac{n!}{(n-3)!} = \frac{2 \times 3 \times 7 \times 5 \times 1 \times 4 \times 6}{1 \times 4 \times 6}$$

$$\frac{n!}{(n-3)!} = \frac{7!}{4!}$$

$$\frac{n!}{(n-3)!} = \frac{7!}{(7-3)!}$$

$$^nP_3 = {}^7P_3$$

$$\boxed{n = 7}$$

(ix) If S = {1, 2, 3, -----, 9}, Events A = {2, 4, 6, 8}, B = {1, 3, 5}, find $P(A \cup B)$.

Ans $S = \{1, 2, 3, ----, 9\}$

$n(S) = 9$

$A = \{2, 4, 6, 8\}$

$B = \{1, 3, 5\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$

$n(A \cup B) = 7$

$$\therefore \quad P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{7}{9}$$

(x) Prove that 1 + 5 + 9 + ---- + (4n – 3) = n (2n – 1), for n = 1, and n = 2.

Ans For $n = 1$,

$$L.H.S = R.H.S = 1$$

For $n = 2$,

$$L.H.S = R.H.S = \frac{3}{2}$$

(xi) Expand up to three terms $(1 - x)^{1/2}$.

Ans $(1 - x)^{1/2} = 1 + \left(\frac{1}{2}\right)(-x) + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)}{2!}(-x)^2 + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right)}{3!}(-x)^3 + - - - -$

$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - - - - - - -$ valid if $|x| < 1$.

(xii) Using binomial theorem, calculate $(0.97)^3$.

Ans $(0.97)^3 = (1 - 0.03)^3$

$= \binom{3}{0}(1)^3(-0.03)^0 + \binom{3}{1}(1)^2(-0.03)^1 + \binom{3}{2}(1)^1(-0.03)^2 + \binom{3}{3}(1)^0(-0.03)^3$

$= 1 + 3 \times (-0.03) + 3 \times (0.0009) - 1 \times 0.000027$

$= 1 - 0.09 + 0.0027 - 0.000027 = 0.9127$

4. Write short answers to any NINE (9) questions: (18)

(i) Find l, when $\theta = \pi$ radians $r = 6$ cm.

Ans As we know that

$l = r\theta$

By putting the given values, we get

$l = 6\pi$

$l = 18.85$ cm

(ii) Verify $\cos 2\theta = 2\cos^2\theta - 1$, when $\theta = 30^\circ, 45^\circ$

Ans $\cos 2\theta = 2\cos^2\theta - \sin^2\theta - \cos^2\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

When $\theta = 30^\circ$

$\cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ$

$$\frac{1}{2} = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$\frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

So, it is proved that

$\cos 2\theta = 2\cos^2\theta - 1$ when $\theta = 30^\circ$

Again, $\theta = 45^\circ$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 90^\circ = \cos^2 45^\circ - \sin^2 45^\circ$

$0 = \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$0 = \frac{1}{2} - \frac{1}{2} = 0$

Hence it is proved that:

$\cos 2\theta = 2\cos^2\theta - 1$ when $\theta = 45^\circ$

(iii) Prove the identity $\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$.

Ans L.H.S $= \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1-\sin\theta}{\cos\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$

$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)} = \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{\cos\theta}{1+\sin\theta}$

= R.H.S

(iv) If α, β, γ are angles of triangle ABC, then prove that $\tan(\alpha+\beta) + \tan\gamma = 0$.

Ans $\tan(\alpha+\beta) + \tan\gamma = 0$ (i)

$\because$ $\alpha + \beta + \gamma = 180^\circ$

$\alpha + \beta = 180^\circ - \gamma$

Put in (i),

$\tan(180^\circ - \gamma) + \tan\gamma = 0$

$\tan(-\gamma) + \tan\gamma = 0$

$-\tan\gamma + \tan\gamma = 0$

$0 = 0$

L.H.S = R.H.S

(v) Prove that $\cos(\alpha + 45^\circ) = \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$.

Ans $\cos(\alpha + 45^\circ) = \cos\alpha\cos 45^\circ - \sin\alpha\sin 45^\circ$

$= \cos\alpha \cdot \frac{1}{\sqrt{2}} - \sin\alpha \cdot \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$

(vi) **Express 2 sin 5 θ cos 2 θ as sum or difference.**

Ans $2 \sin 5\theta \cos 2\theta = \sin(5\theta + 2\theta) + \sin(5\theta - 2\theta)$

$= \sin 7\theta + \sin 3\theta$

(vii) **Find the period of $\cos \frac{x}{6}$.**

Ans $\cos \frac{x}{6} = \cos\left(\frac{x}{6} + 2\pi\right) = \cos \frac{1}{6}(x + 12\pi)$

Hence period of $\cos \frac{x}{6}$ is 12π.

(viii) **In a right angle triangle ABC, a = 5429, c = 6294 and $\gamma = 90^\circ$. Find b, α.**

Ans Given, $a = 5429$

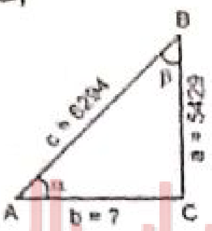
$c = 6294$

$\gamma = 90^\circ$

To find $b = ?$

$\alpha = ?$

From the above data,



From figure,

$$\sin \alpha = \frac{5429}{6294}$$

$$\sin \alpha = 0.862567$$

$$\alpha = \sin^{-1}(0.862567)$$

$$\alpha = 59.606^\circ$$

$$\boxed{\alpha = 59^\circ 36'}$$

And by Pythagora's Theorem

$$c^2 = b^2 + a^2$$

$$c^2 - a^2 = b^2$$

$$\Rightarrow \quad b^2 = c^2 - a^2$$

$$b^2 = (6294)^2 - (5429)^2$$

$$b^2 = 10140395$$

$$\boxed{b = 3184.398}$$

(ix) **Define the term circum-circle.**

Ans The circle passing through the three vertices of a triangle is called a circum-circle.

(x) Find the area of triangle ABC if $a = 524$, $b = 276$, $c = 315$.

Ans Given, $a = 524$, $b = 276$, $c = 315$

$$s = \frac{a+b+c}{2}$$

$$= \frac{524+276+315}{2} = \frac{1115}{2}$$

$$s = 557.5$$

$$s-a = 33.5,\ s-b = 281.5,\ s-c = 242.5$$

By area formula,

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{557.5(33.5)(281.5)(242.5)}$$

$$= 35705.894 \text{ square units}$$

(xi) Show that $\cos(\sin^{-1} x) = \sqrt{1-x^2}$.

Ans Given, $\cos(\sin^{-1} x) = \sqrt{1-x^2}$

$$\text{L.H.S} = \cos(\sin^{-1} x)$$

Let $\sin^{-1} x = \theta$

$$x = \frac{1}{\sin}\theta$$

$$x = \cos\theta$$

As $\cos\theta = \sqrt{1-(\sin\theta)^2}$

$$x = \sqrt{1-(\sin\theta)^2}$$

As $\theta = \sin^{-1}(x)$

$$= \sqrt{1-[\sin(\sin^{-1}(x))]^2}$$

As $\theta = \sin[\sin^{-1}(\theta)] = \sqrt{1-x^2} = \text{R.H.S}$

(xii) Find solutions of $\operatorname{cosec}\theta = 2$, $\theta \in [0, 2\pi]$.

Ans $\operatorname{cosec}\theta = 2$

or $\dfrac{1}{\operatorname{cosec}\theta} = \dfrac{1}{2}$

$\Rightarrow$ $\sin\theta = \dfrac{1}{2}$

$\therefore$ $\sin\theta$ is positive in first and second quadrants with the angle $\theta = \frac{\pi}{6}$.

$\therefore$ $\theta = \dfrac{\pi}{6}$

and $\theta = \pi - \dfrac{\pi}{6}$

$$\theta = \frac{5\pi}{6}$$

$$\frac{\pi}{6}, \frac{5\pi}{6}$$

(xiii) Solve $2\sin\theta + \cos^2\theta - 1 = 0,\ \theta \in [0, \pi]$.

Ans

$$2\sin\theta + \cos^2\theta - 1 = 0$$
$$2\sin\theta - (1 - \cos^2\theta) = 0$$
$$2\sin\theta - \sin^2\theta = 0$$
$$\sin\theta\,(2 - \sin\theta) = 0$$

$\therefore\ \sin\theta = 0$	;	$2 - \sin\theta = 0$
$\theta = \sin^{-1} 0$	;	$2 = \sin\theta$
$\theta = 0, \pi$	;	impossible as $\lvert\sin\theta\rvert \le 1$

Thus, the answer will be $0, \pi$.

SECTION-II

NOTE: Attempt any Three (3) questions.

Q.5.(a) Give the logical proof of De Morgan's Laws. (5)

Ans (i) $(A\cup B)' = A'\cap B'$

Let $x \in (A\cup B)'$

$\Rightarrow\quad x \notin A\cup B$

$\Rightarrow\quad x \notin A$ and $A \notin B$

$\Rightarrow\quad x \in A'$ and $x \in B'$

$\Rightarrow\quad x \in A'\cap B'$ (1)

But x is an arbitrary member of $(A\cup B)'$

Therefore, (1) means that $(A\cup B)' \subseteq A'\cap B'$ (2)

Now suppose that $y \in A'\cap B'$

$\Rightarrow\quad y \in A'$ and $y \in B'$

$\Rightarrow\quad y \notin A$ and $y \notin B$

$\Rightarrow\quad y \notin A\cup B$

$\Rightarrow\quad y \in (A\cup B)'$

Thus $A'\cap B' \subseteq (A\cup B)'$ (3)

From (2) and (3), we conclude that

$$(A\cup B)' = A'\cap B'$$

(ii) $(A\cap B)' = A'\cup B'$

It may be proved similarly or deducted from $A \cup B = B \cup A$ by complementation

(iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Let $x \in A \cup (B \cap C)$

$\Rightarrow \quad x \in A$ or $x \in B \cap C$

$\Rightarrow \quad$ If $x \in A$ it must belong to $A \cup B$ and $x \in A \cup C$

$\Rightarrow \quad x \in (A \cup B) \cap (A \cup C) \quad (1)$

Also if $x \in B \cap C$, then $x \in B$ and $x \in C$.

$\Rightarrow \quad x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow \quad x \in (A \cup B) \cap (A \cup C)$

Thus $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \quad (2)$

Conversely, suppose that

$y \in (A \cup B) \cap (A \cup C)$

There are two cases to consider:

$y \in A, y \notin A$

In the first case, $y \in A \cup (B \cap C)$

If $y \notin A$, it must belong to B as well as C

i.e., $y \in (B \cap C)$

$\therefore \quad y \in A \cup (B \cap C)$

So in either case,

$y \in (A \cup B) \cap (A \cup C) \Rightarrow y \in A \cup (B \cap C)$

Thus $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \quad (3)$

From (2) and (3), it follows that

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

It may be proved similarly or deducted from

$A \cup (B \cup C) = (A \cup B) \cup C$

by complementation.

(b) Prove that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$

Ans L.H.S = $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix}$

Adding C_2 in C_1, we get

$$= \begin{vmatrix} a+b+c & a & a^2 \\ a+b+c & b & b^2 \\ a+b+c & c & c^2 \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

By interchanging rows and columns,

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

By $C_2 - C_1$, $C_3 - C_1$,

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

Expanding by R_1,

$$= (a+b+c)\begin{vmatrix} b-a & c-a \\ b^2-a^2 & c^2-a^2 \end{vmatrix}$$

By taking common: $b-a$ from C_1 and $c-a$ from C_2

$$= (a+b+c)(b-a)(c-a)\begin{vmatrix} 1 & 1 \\ b+a & c+a \end{vmatrix}$$

$$= (a+b+c)(b-a)(c-a)\,[1(c+a) - 1(b+a)]$$
$$= (a+b+c)(b-a)(c-a)(c-b)$$
$$= (a+b+c)(-1)(a-b)(c-a)(-1)(b-c)$$
$$= (a+b+c)(a-b)(b-c)(c-a)$$

= R.H.S Proved

Q.6.(a) Show that the roots of $(mx+c)^2 = 4ax$ will be equal, if $c = \frac{a}{m}$; $m \neq 0$. (5)

Ans

$(mx+c)^2 = 4ax$

$m^2x^2 + 2(mc - 2a)x + c^2 = 0$

$= b^2 - 4ac$

$= [2(mc-2a)]^2 - 4(m^2)(c^2)$

$= 4(m^2c^2 + 4a^2 - 4am) - 4m^2c^2$

$= 4m^2c^2 + 16a^2 - 16amc - 4m^2c^2$

$= 16a[a - mc]$

roots, will be equal if disc = 0

i.e., $16a[a-mc] = 0 \Rightarrow a - mc = 0$

$\Rightarrow \quad c = \frac{a}{m}, \quad m \neq 0$

(b) Resolve into partial fractions of $\frac{2x+1}{(x-1)(x+2)(x+3)}$ (5)

Ans

$$\frac{2x+1}{(x-1)(x+2)(x+3)}$$

Let,

$$\frac{2x+1}{(x-1)(x+2)(x+3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3} \qquad \text{(i)}$$

Multiply by $(x-1)(x+2)(x+3)$ on both sides

$$2x + 1 = A(x+2)(x+3) + B(x-1)(x+3) + C(x-1)(x+2) \qquad \text{(ii)}$$

Put $x = 1$ in equation (ii), we have

$$2(1) + 1 = A(1+2)(1+3)$$
$$2 + 1 = A(3)(4)$$
$$\frac{3}{(3)(4)} = A$$
$$\Rightarrow \quad \boxed{A = \frac{1}{4}}$$

Put $x = -2$ in equation (ii), we have

$$2(-2) + 1 = B(-2-1)(-2+3) + B(0) + C(0)$$
$$-4 + 1 = B(-3)(+1)$$
$$-3 = -3B$$
$$\Rightarrow \quad \boxed{B = 1}$$

Put $x = -3$ in equation (ii), we have

$$2(-3) + 1 = C(-3-1)(-3+2)$$
$$-6 + 1 = C(-4)(-1)$$
$$-5 = 4C$$
$$\Rightarrow \quad \boxed{C = \frac{-5}{4}}$$

Putting the values of A, B and C in equation (i), we have

$$\frac{2x+1}{(x-1)(x+2)(x+3)} = \frac{1}{4x-1} + \frac{1}{x+2} - \frac{5}{4(x+3)}$$

Hence partial fractions are

$$\frac{1}{4(x-1)} + \frac{1}{x+2} - \frac{5}{4(x+1)}$$

Q.7.(a) If a, b, c, d are in G.P., prove that $a^2 - b^2, b^2 - c^2, c^2 - a^2$ are in G.P. **(5)**

Ans If r is the common ratio of the G.P. a, b, c, d

$$r = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

$\therefore$ $b = ar$ (i)

$c = br = ar^2$ (ii)

$$d = cr = ar^3 \qquad \text{(iii)}$$

Now $a^2 - b^2, b^2 - c^2, c^2 - d^2$ will be in G.P.

if $\dfrac{b^2 - a^2}{a^2 - b^2} = \dfrac{c^2 - d^2}{b^2 - c^2}$

or if $(b^2 - c^2)^2 = (a^2 - b^2)(c^2 - d^2)$

By using (i) and (ii), we have

$$\text{L.H.S} = (b^2 - c^2)^2 = (a^2r^2 - a^2r^4)^2$$
$$= a^4r^4(1 - r^2)^2$$
$$\text{R.H.S} = (a^2 - b^2)(c^2 - d^2)$$

By using (i), (ii) and (iii),

$$= (a^2 - a^2r^2)(a^2r^4 - a^2r^6)$$
$$= a^2(1 - r^2)a^2r^4(1 - r^2)$$
$$= a^4r^4(1 - r^2)^2$$

As L.H.S = R.H.S

So, $a^2 - b^2, b^2 - c^2, c^2 - d^2$ are in G.P.

(b) Find the term involving x^4 in the expansion of $(3 - 2x)^7$. (5)

Ans Let T_{r+1} be the required. Then

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$
$$= \binom{7}{r} 3^{7-r}(-2x)^r$$
$$= \binom{7}{r} 3^{7-r}(-2)^r(x)^r \qquad \text{(i)}$$

For the term involving x^4, put exponent of x equal to 4, *i.e.*, $r = 4$

$$T_{4+1} = \binom{7}{4} 3^{7-4}(-2)^4 x^4$$
$$T_5 = \frac{7 \times 6 \times 5}{3 \times 2 \times 1}(3^3)(16)x^4$$
$$= 15120\,x^4$$

Q.8.(a) Prove the identity: (5)

$$\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

Ans
$$\text{L.H.S} = \frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta}$$
$$= \frac{\sin\theta - (\operatorname{cosec}\theta - \cot\theta)}{\sin\theta(\operatorname{cosec}\theta - \cot\theta)}$$

$$= \frac{\sin\theta - \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}{\sin\theta\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)} = \frac{\frac{\sin^2\theta - 1 + \cos\theta}{\sin\theta}}{\sin\theta\left(\frac{1-\cos\theta}{\sin\theta}\right)}$$

$$= \frac{\sin^2\theta - 1 + \cos\theta}{\sin\theta\,(1-\cos\theta)} = \frac{1 - \cos^2\theta - 1 + \cos\theta}{\sin\theta\,(1-\cos\theta)}$$

$$= \frac{\cos\theta\,(1-\cos\theta)}{\sin\theta\,(1-\cos\theta)}$$

$$\text{L.H.S} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

Now $\text{R.H.S} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

$$= \frac{\operatorname{cosec}\theta + \cot\theta - \sin\theta}{\sin\theta\,(\operatorname{cosec}\theta + \cot\theta)}$$

$$= \frac{\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} - \sin\theta}{\sin\theta\left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)} = \frac{\frac{1 + \cos\theta - \sin^2\theta}{\sin\theta}}{\sin\theta\left(\frac{1+\cos\theta}{\sin\theta}\right)}$$

$$= \frac{1 + \cos\theta - \sin^2\theta}{\sin\theta\,(1+\cos\theta)} = \frac{\cos\theta + 1 - \sin^2\theta}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{\cos\theta + \cos^2\theta}{\sin\theta\,(1+\cos\theta)} = \frac{\cos\theta\,(1+\cos\theta)}{\sin\theta\,(1+\cos\theta)}$$

$$\text{R.H.S} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

Hence $\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

(b) Prove the identity $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$. (5)

Ans $\text{L.H.S} = \frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta}$

$$= \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin(3\theta - \theta)}{\sin\theta\cos\theta} = \frac{\sin 2\theta}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2 = \text{R.H.S}$$

Q.9.(a) Prove that in an equilateral triangle, $r : R : r_1 = 1 : 2 : 3$. (5)

Ans As in equilateral triangle, all sides are equal so we take

$$a = b = c$$

Then $s = \frac{a+a+a}{2}$ $\left(\text{As } s = \frac{a+b+c}{2}\right)$

$$s = \frac{3a}{2}$$

Now $s-a = s-b = s-c$ (As all sides are equal)

$$= \frac{3a}{2} - a = \frac{3a-2a}{2} = \frac{a}{2}$$

Now $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{\left(\frac{3a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)} = \sqrt{\frac{3a^4}{4\times 4}} = \frac{\sqrt{3}\,a^2}{4}$$

Now $r = \frac{\Delta}{s} = \frac{\sqrt{3}\,a^2}{4} \div \frac{3a}{2}$ $\left(\text{As } s = \frac{3a}{2}\right)$

$$r = \frac{\sqrt{3}\,a^2}{4} \times \frac{2}{3a} = \frac{\sqrt{3}\,a}{6} = \frac{\sqrt{3}\,a}{3\times 2} = \frac{a}{2\sqrt{3}}$$

$$\Rightarrow \quad \boxed{r = \frac{a}{2\sqrt{3}}}$$

Now $R = \frac{abc}{4\Delta} = \frac{(a)(a)(a)}{4\frac{\sqrt{3}\,a^2}{4}} = \frac{a^3}{\sqrt{3}\,a^2}$

$$\boxed{R = \frac{a}{\sqrt{3}}}$$

As $r_1 = \frac{\Delta}{s-a} = \frac{\sqrt{3}\,a^2}{4} \div \left(\frac{a}{2}\right)$

$$= \frac{\sqrt{3}\,a^2}{4} \times \frac{2}{a} = \frac{\sqrt{3}\,a}{2}$$

$$\Rightarrow \quad \boxed{r_1 = \frac{\sqrt{3}\,a}{2}}$$

Now L.H.S $= r : R : r_1$

Putting values of r, R and r_1

$$= \frac{a}{2\sqrt{3}} : \frac{a}{\sqrt{3}} : \frac{\sqrt{3}\,a}{2}$$

$\div$ by a

$$= \frac{1}{2\sqrt{3}} : \frac{1}{\sqrt{3}} : \frac{\sqrt{3}}{2}$$

Multiplying by $\sqrt{3} \times 2$

$$= 2\sqrt{3} \times \frac{1}{2\sqrt{3}} : \sqrt{3} \times 2 \times \frac{1}{\sqrt{3}} : \sqrt{3} \times 2 \times \frac{\sqrt{3}}{2}$$

$$= 1 : 2 : \sqrt{9} = 1 : 2 : 3$$

So proved $\boxed{r : R : r_1 = 1 : 2 : 3}$

(b) Prove that $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1} 3 = \frac{\pi}{4}$. **(5)**

Ans Let $x = \sin^{-1}\frac{1}{\sqrt{5}}$

$$\sin x = \frac{1}{\sqrt{5}}$$

$$\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \left(\frac{1}{\sqrt{5}}\right)^2}$$

$$\cos x = \sqrt{\frac{5-1}{5}}$$

$$\cos x = \frac{2}{\sqrt{5}} \qquad y = \cot^{-1} 3 \qquad \cot y = 3$$

$$\operatorname{cosec} y = \sqrt{1 + \cot^2 y} = \sqrt{1 + (3)^2}$$

$$\operatorname{cosec} y = \sqrt{10} \quad \sin y = \frac{1}{\sqrt{10}}$$

$$\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - \left(\frac{1}{10}\right)^2} = \sqrt{\frac{10-1}{10}}$$

$$\cos y = \frac{3}{\sqrt{10}}$$

Using

$$\sin(x + y) = \sin x \cdot \cos y + \cos x \cdot \sin y$$

$$= \frac{1}{\sqrt{5}} \cdot \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{10}} = \frac{3+2}{\sqrt{50}} = \frac{5}{5\sqrt{2}}$$

$$x + y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1} 3 = \frac{\pi}{4}$$